



**RESEARCH PAPER**

**Public-Private Investment Sustenance: A Glance of Education Sector of Pakistan**

**<sup>1</sup>Muhammad Rizwan Khursheed, <sup>2</sup>Khawaja Asif Mehmood\* and <sup>3</sup>Muhammad Nasir Hussain**

1. MPhil Business Economics Scholar, School of Economics, Bahauddin Zakariya University, Multan, Pakistan
2. Assistant Professor of Economics, School of Economics, Bahauddin Zakariya University, Multan, Pakistan
3. PhD Economics Scholar, Xi'an Jiaotong University, China

**\*Corresponding Author:** khawjaasif@bzu.edu.pk

**ABSTRACT**

The public and private sector are essential to materialize the national plans which are in the interest of the general public. The underlying study is concerned with the sustainability of governmental and private partnership, notably in Pakistan's education sector. The data is ranged from 1980 to 2022. For the estimation of results, Auto Regressive Distributive Lag (ARDL) approach is used. The variables considered in this study are: Private Education Expenditure (PED), Economic Growth (EGR), Government Education Expenditure (GED), Poverty (PVR), Population Growth (PGR), Inflation (INF), and Personal Remittances (REM). A mixed connection between public and private sector (PPS) in the education sector is found. In the long run, significant and positive relationship is found between PED and GED. However, in case of short run, the results are dissimilar. The policy recommendations urge the ongoing public and private sector investment in the education sector for the best interest of the nation.

**KEYWORDS** ARDL, Education, Pakistan, Public & Private Sector

**Introduction**

Infrastructure development in the nation is significantly influenced by PPS. The sustainability impact between the public and private sectors, notably in the education sector, is examined in this study. Basically, neither the public nor the private sectors can build the education infrastructure alone. On the other hand, PPS has helped to raise the standard of education. Establishing schools, colleges, and universities has been made possible by collaboration between the public and private sectors. PPS has several benefits for the educational sector but there are also many downsides in this strategy. One of these hitches is the potential for profit-seeking over providing services by private organizations which might lead to a conflict of interest with the government.

The government of Pakistan strongly supports PPS, having launched 108 such projects within 1990 to 2019, attracting a total investment of \$28.4 billion. In early 2021, an amendment to the PPS legislation further reinforced this commitment, expanding its scope to include both physical infrastructure and social sector initiatives, particularly in the realm of education. PPS has played a crucial role in transforming education sector. Although the initial private university was established in 1983, its growth was limited. By 1990s that the government introduced PPS into the education system and it went to be a turning point in enhancing the growth and quality of education sector in Pakistan. One noteworthy example of PPS in Pakistan is the Punjab Education Foundation which facilitates affordable education through a partnership between the government of Punjab and non-governmental organizations.

According to the Ministry of Finance, PPS is vital for improving public education by increasing access to quality education, particularly in underserved areas. PPS involves private institutions contributing resources, expertise, and technology. It also addresses workforce skills gaps through vocational training. However, the IMF stresses the need for careful design and execution. Private sector partners should share costs and risks while the public sector retains control. The focus should remain on essential services, and rigorous evaluation and monitoring are crucial to ensure PPS delivers as expected.

PPS has gained popularity in Pakistan as a means of financing and developing infrastructure projects. Yet, certain often overlooked or undervalued issues include limited access to financing, project viability, risk distribution, institutional capacity constraints, the legal framework, and transparency shortcomings.

The problem lies in the absence of a strong regulatory framework for PPS in Pakistan's education sector, leading to innovation gaps and project inefficiency. To address challenges like low literacy rates and poor infrastructure, various initiatives, such as school upgrades, digitization, and teacher capacity building, are being pursued at provincial and federal levels to fulfill Sustainable Development Goals.

According to data from the Ministry of Finance, enrollment figures in the education sector have shown significant growth: Pre-primary education increased from 13.5 million students in 2019-2020 to 14.4 million in 2020-21. Primary education witnessed a rise from 23.6 million in 2018-19 to 25.7 million in 2020-21. Middle education enrollment went up from 7.6 million to 8.3 million during 2018-19 and 2020-21. Secondary schools experienced an increase from 4.0 million students in 2018-19 to 4.5 million in 2020-21. Higher secondary schools have pledged enrollment growth from 2.14 million in 2018-19 to an anticipated 2.55 million in 2020-21. Technical and vocational institutes had an enrollment increase from 0.43 million to an expected 0.50 million in 2020-21. Degree colleges are projected to have 0.76 million students in 2020-21, up from 0.74 million in 2019-20. Finally, universities are also expected to enroll 1.96 million students in 2020-21, showing growth from 1.91 million in 2019-20.

PPS, such as joint ventures and concessions, blend public funding and private expertise, fostering efficient collaborations. In infrastructure, this approach results in cost-effective, high-quality projects for the public.

However, the novelty of study is to locate if the private sector which aims at profit making takes initiative to invest in education sector after there is pledged heavy funds by the public sector.

## **Literature Review**

This study examines the collaboration between public and private investment sustainability in education sector. Fewer researches highlight the importance of this study area. The researches supporting these efforts over the past three decades include Bano (2008), Farooq et al. (2017), Awan and Waqar (2019), Farah and Rizvi (2007), Halai (2011), Irfan (2015), Verger et al. (2017), Khalid et al. (2016), Maluleke et al. (2023), and Hassan et al. (2011), and Rind (2013) who have examined various aspects of public-private sector investment in educational infrastructure, funding, and financial support.

Bano (2008) analyzed that Pakistan's shift towards PPS aimed to enhance education access and equity. However, ad hoc programs relying on NGOs and funders, trust issues, and insufficient incentives hindered PPS progress. Surveys conducted from 2003 to 2007 showed that students, especially in mathematics, often didn't meet grade-level expectations. Low-income families faced challenges as non-formal equity programs lacked clear pathways to higher education, exacerbated by the absence of nearby secondary schools.

Farooq et al. (2017) conducted a study on Pakistan's preschool system and discovered a concerning statistic: 22.6 million children, equivalent to 44% of Pakistani children, were not enrolled in school. Their research focused on social connections, education, and teaching experience to establish a three-factor model for intellectual capital in Pakistan. The study confirmed the model's relevance in Pakistan's secondary schools and highlighted structural capital as a significant influencing factor. The study also used the dependent variable, intellectual capital, to analyze gender differences.

Awan and Waqar (2019) investigated the impact of the expanding private school sector on literacy rates, utilizing data from Pakistan's Economic Survey spanning from 2000 to 2015. Their findings revealed that private schools outperformed public ones in terms of literacy rates. They also identified that private partnership had a significant influence on the growth of literacy and the development of human capital. The study attributed improvements in academic performance to the growth of private education in Pakistan, driven by dissatisfaction with the deficiencies and corruption within the public education system.

Farah and Rizvi (2007) explored Pakistan's early PPS approach in education during the 1990s. They found that PPS models were successful in improving underprivileged elementary schools, demonstrating their effectiveness. However, these partnerships often faced challenges in terms of long-term sustainability. The primary aim was to enhance education opportunities for underrepresented populations.

Halai (2011) investigated the emergence of around 60 new private universities in Pakistan since 1983, greatly expanding higher education access. The Higher Education Commission, established in 2002, played a crucial oversight role. Post-2002, student enrollment surged to 800,000 by 2009, with 115,000 in private universities. This boosted access for 18 to 23-year-olds to 4.7%. Yet, accommodating 1.3 million tertiary-level students by 2010 required substantial resources.

Irfan (2015) investigated PPS involvement with the collaboration of various stakeholders to achieve common goals and are prevalent worldwide. This research focused on specific PPS initiatives in Pakistan's education sector, specifically the adoption of public schools and partnerships with not-for-profit organizations. The study collected and analyzed data, employing case studies and comprehensive literature reviews to address the challenges associated with these initiatives.

Rind (2022) explored the increasing adoption of PPS for educational reform in developing countries, particularly in Pakistan, which is emphasized. PPSs entail the private sector's involvement in education through legal agreements to achieve educational objectives while sharing risks. The study evaluated the success of PPS in Sindh, Pakistan, highlighting the utilization of low paid contract workers for cost efficiency. Study of Rind (2022) served as a valuable resource for policymakers, stakeholders and researchers interested in PPS driven educational reforms.

MacPhail (2013) explored the impact of PPS on educational governance. The book emphasizes the connection between education and the economy, introducing a new framework for educational administration and policy. It critically evaluates the theoretical foundations and outcomes of PPS implementation, offering insights into how PPS has influenced domestic education systems in developed nations as its popularity continues to grow.

Verger et al. (2017) examined the substantial for-profit global education market. The research encompassed three cases: its influence on international education policy, the role of charter schools, and the presence of low-cost private schools in the Global South.

Drawing on Burch's work from 2009, the study emphasized how networks serve as efficient channels and informal governing systems across various industries, including education.

Khalid et al. (2016) focused on how PPS contribute to the development of a nation's school systems. Khalid et al. (2016) concluded that PPS, which entails collaboration between the public and private sectors, is an effective strategy for enhancing resource availability. The study specifically examined the Punjab Education Foundation affiliated schools in Pakistan's District Bhakkar as its subject.

Likewise, Hassan et al. (2011) investigated the public and private investment sustainability in Malaysia by the mean of panel regression. The findings were satisfactorily favoring the complementary posit of public and private investment in varied sectors of Malaysia. On the similar note, public private investment was found to determine one another in the study of Maluleke et al. (2023).

### **Material and Methods**

The methodological issues are discussed that enable to meet the objectives of the study. For the analytical purpose, the key variables concentrated upon are including Private Expenditure on Education (PED), Economic Growth (EGR), Government Education Expenditure (GED), Poverty (PVR), Population Growth (PGR), Inflation (INF), and Personal Remittances (REM), to analyze PPS's effectiveness in improving access to quality education.

### **Data Collection**

In this study, time-series data gathered for Pakistan from 1980 to 2022, by focusing on seven macroeconomic variables: Private Expenditure on Education, Economic Growth, Government Education Expenditure, Poverty, Population Growth, Inflation, and Personal Remittances. Data sources include Ministry of Finance (Government of Pakistan), Pakistan Bureau of Statistics (PBS), Ministry of Pakistan, Prime Minister Advisory Wing, Government of Pakistan, World Development Indicator, and State Bank of Pakistan's Handbook of Statistics 2022.

### **Conceptual Framework**

PPS in Pakistan aims to enhance accessibility, infrastructure, and educational quality. Ensuring adequate incentives for private sector partners is crucial for sustainability, and the research in this context is guided by relevant literature.

Irfan (2015) investigated the relationship between various partners in PPS, where resources are pooled to achieve common goals. Primary focus was on PPS within Pakistan's education sector. Farooq et al. (2017) uncovered a concerning situation in Pakistan's primary education, revealing that 44% of Pakistani children, equivalent to 22.6 million, were not attending school. Similarly, Halai (2011) highlighted the amazing expansion of private education in Pakistan, noting the emergence of more than 60 private universities since the first private university's founding in 1983. Whereas, Bano (2008) examined the application of PPS strategies within Pakistan's government education plan to tackle issues related to accessibility, quality, and fairness. Meanwhile, Farah and Rizvi (2007) delved into how the Pakistani government promoted PPS during the early 1990s to enhance education in underprivileged primary schools. Furthermore, Awan and Waqar (2019) uncovered that Pakistan's private education system significantly boosts literacy rates, especially in primary and higher education. Whilst, Khalid et al. (2016) examined how PPS contribute to enhancing school infrastructure in developing countries, tackling financial and infrastructure obstacles. On the similar note, Verger et al. (2017) explored the global education sector, emphasizing its growth in significant transactional markets and offering examples of its functioning.

After a thorough review of current literature, this study puts forth the following model to align with its research objectives.

$$PED = f(EGR, GED, PVR, PGR, INF, REM)$$

The equation characterizes the affiliation between the dependent variable, private education expenditure, and various independent variables such as; Economic Growth, Government Education Expenditure, Poverty, Population Growth, Inflation, and Personal Remittances.

$$PED_t = \beta_0 + \beta_1 EGR_t + \beta_2 GED_t + \beta_3 PVR_t + \beta_4 PGR_t + \beta_5 INF_t + \beta_6 REM_t + \varepsilon_t$$

In this equation,  $\beta_1, \beta_2, \dots, \beta_6$  represents the numerical coefficients of  $\beta_0$  derived from the model, while  $\varepsilon_t$  signifies the error term.

### **Description of Variables**

The descriptions of the variables are given below:

#### **Private Expenditure on Education (PED)**

The share of GDP dedicated to private education refers to spending on education by households and private institutions, covering costs like tuition, books, uniforms, and transport. It excludes government education spending and can be expressed as a percentage of GDP.

#### **Economic Growth (EGR)**

The annual GDP growth percentage is determined by using stable local currency values and aggregates based on constant 2015 prices, denominated in U.S. dollars.

#### **Government Education Expenditure (GED)**

The share of GDP allocated to education indicates present education spending. The variable is measured in in PKR.

#### **Poverty (PVR)**

The national poverty headcount ratio is a measure indicating the percentage of people living below the poverty line established at the national level, based on data gathered from household surveys.

#### **Population Growth (PGR)**

The annual population growth rate for a given year (t) is determined by quantifying the exponential surge in population between the year's midpoint and the prior year (t-1). This increase is expressed as a percentage and is grounded in the comprehensive population definition, encompassing all inhabitants, irrespective of their citizenship or legal status.

#### **Inflation (INF)**

The annual percentage GDP growth rate, as calculated at market prices, is ascertained through the utilization of consistent local currency values and aggregate figures derived from 2015 price levels, which are subsequently expressed in United States dollars.

### **Personal Remittances (REM)**

Personal remittances received (Current US\$) represents the aggregate monetary inflow into a country originating from individuals who are employed or living abroad. This encompasses financial transfers facilitated through both formal and informal channels.

### **Empirical Analysis**

Empirical analysis is a research method that employs data and measurements to validate theories and hypotheses about a phenomenon or topic. The empirical methods of analyses are given below:

#### **Unit Root Test**

A unit root test detects non-stationary trends in time series data for research precision. The Dickey-Fuller test, introduced in 1979, is common, and the Augmented Dickey-Fuller (ADF) test, an enhanced version, is widely used, as highlighted in studies by Mehmood and Hassan (2015).

$$\Delta y_t = \alpha_1 + \alpha_2 t + \delta y_{t-1} + \beta_t \sum_{i=1}^m \Delta y_{t-i} + \mu_t$$

Where  $\mu_t$  is residual term (white noise),  $y_t$  sequential predictors,  $\delta$  explains that there is no change and

$$\Delta y_{t-1} = (y_{t-1} - y_{t-2}), \Delta y_{t-2} = (y_{t-2} - y_{t-3})$$

When the test statistic is less than its critical value, the variable is classified as stationary.

#### **Bounds Test of Cointegration**

The ARDL bounds testing method, established by Pesaran et al. (2001), is a statistical technique used to evaluate long-term correlations among multiple time series variables.

#### **Autoregressive Distributed Lag**

The ARDL model is a versatile regression approach for examining relationships between variables over both short and long periods, especially when working with time-series data and their interdependencies.

#### **Test of Long Run Relationship (Cointegration)**

This study examines long-term correlations between variables through unit root tests to ascertain stationarity. In cases of different integration levels (e.g., I(0) or I(1)), the ARDL approach validates cointegration, offering a more effective relationship assessment. An un restricted Error Correction Model (ECM) establishes enduring connections guided by the model's functional structure.

$$\begin{aligned} \Delta PED_t &= \alpha + \beta_1 PED_{t-1} + \beta_2 EGR_{t-2} + \beta_3 GED_{t-3} + \beta_4 PVR_{t-4} + \beta_5 PGR_{t-5} + \beta_6 INF_{t-6} + \beta_7 REM_{t-7} \\ &+ \sum_{l=0}^{P_1} \delta_1 PED_{t-l} + \sum_{l=0}^{P_2} \delta_2 EGR_{t-l} + \sum_{l=0}^{P_3} \delta_3 GED_{t-l} + \sum_{l=0}^{P_4} \delta_4 PVR_{t-l} + \sum_{l=0}^{P_5} \delta_5 PGR_{t-l} + \sum_{l=0}^{P_6} \delta_6 INF_{t-l} \\ &+ \sum_{l=0}^{P_7} \delta_7 REM_{t-l} + \varepsilon_t \end{aligned}$$

Based on the anticipated coefficients in a regression model that illustrate the long-term relationship between variables, where,  $\beta_1$ ,  $\delta_1$  and  $\Delta$  are the long run multipliers. However,  $\varepsilon_t$  is error term's white noise counterpart, while on the other hand, it indicates randomness and independence. To determine the coefficients of all variables in the presence of a long-run relationship, one must adhere to the equations of the models.

$$\begin{aligned} \Delta PED_t &= \alpha + \sum_{l=0}^{P_1} \eta_1 \Delta PED_{t-l} + \sum_{l=0}^{P_2} \eta_2 \Delta EGR_{t-l} + \sum_{l=0}^{P_3} \eta_3 \Delta GED_{t-l} + \sum_{l=0}^{P_4} \eta_4 \Delta PVR_{t-l} + \sum_{l=0}^{P_5} \eta_5 \Delta PGR_{t-l} + \sum_{l=0}^{P_6} \eta_6 \Delta INF_{t-l} \\ &+ \sum_{l=0}^{P_7} \eta_7 \Delta REM_{t-l} + \varepsilon_t \end{aligned}$$

### ARDL Short run Relationship

Having established long-term relationships between chosen variables, the approach then delves into short-term connections using the Error Correction Model (ECM). The specific ECM equations are as follows:

$$\begin{aligned} \Delta PED_t &= \alpha + \sum_{l=0}^{P_1} \lambda_1 \Delta PED_{t-l} + \sum_{l=0}^{P_2} \lambda_2 \Delta EGR_{t-l} + \sum_{l=0}^{P_3} \lambda_3 \Delta GED_{t-l} + \sum_{l=0}^{P_4} \lambda_4 \Delta PVR_{t-l} + \sum_{l=0}^{P_5} \lambda_5 \Delta PGR_{t-l} + \sum_{l=0}^{P_6} \lambda_6 \Delta INF_{t-l} \\ &+ \sum_{l=0}^{P_7} \lambda_7 \Delta REM_{t-l} + \delta ECM + \varepsilon_t \end{aligned}$$

Where  $\lambda$  shows short-run variables and  $\delta$  is the coefficient of ECM that reflect the speed of adjustment in the equilibrium.

### Sensitivity Analysis

The reliability of statistical tests and models can be evaluated using sensitivity analysis, which is a vital for confirming the authenticity of the results. In this study, tests including the Breusch-Pagan-Godfrey LM, heteroskedasticity, RAMSEY RESET, histogram normality, and CUSUM are critically analyzed to understand their robustness and limitations. The Breusch-Pagan-Godfrey LM test's sensitivity to changes in regression model specifications and data choice, for instance, is assessed. It also evaluates how changes to the parameters for detecting heteroskedasticity affect the heteroskedasticity test. Sensitivity analysis is a useful tool for doing statistical analyses that result in findings those are relied upon, allowing for more informed decision making.

### Results and Discussions

This Section's key goal is to present pivotal findings on PPS sustainability in Pakistan's education sector.

**Descriptive Analyses**

**Table 1**  
**Descriptive Analyses Results**

	<b>PED</b>	<b>EGR</b>	<b>GED</b>	<b>PVR</b>	<b>PGR</b>	<b>INF</b>	<b>REM</b>
<b>Mean</b>	0.87	4.70	3.10	24.30	2.58	8.20	7.69
<b>Maximum</b>	3.88	10.22	9.84	34.60	4.42	20.29	31.31
<b>Minimum</b>	-0.23	-1.27	2.65	17.32	1.20	2.53	1.00
<b>Std. Dev.</b>	1.04	2.21	2.83	3.60	0.85	3.68	8.28
<b>Skewness</b>	1.45	-0.14	1.07	1.20	0.14	0.66	1.21
<b>Kurtosis</b>	4.22	3.29	2.86	4.66	2.16	3.91	3.24
<b>Jarque-Bera</b>	17.68	0.29	8.19	15.23	1.41	4.62	10.62
<b>Probability</b>	0.0001	0.87	0.02	0.0005	0.4952	0.1	0.005

It appears that the central value of PED with a maturity of (3 months), (6 months) and (9 months) is 0.87 percent, the average value of EGR is 4.70 percent, the value of average GED is 3.10 percent, INF is 8.20 percent, PGR is 2.58 percent, PVR is 24.30 percent and the value of REM is 7.69 percent between the sample period of 1980 to 2022. The maximum range of PED in our sample period is 3.88, the maximum value of EGR is 10.22, GED is 9.84, PVR is 34.60, PGR is 4.42, INF is 20.29 and the maximum value of REM is 31.31 and the minimum range of PED is -0.23, EGR is -1.27, GED is 2.65, PVR is 17.32, PGR is 1.20, INF is 2.53 and the minimum value of REM is 1.00.

Standard deviation which means the difference of individual value from the mean value of PED is 1.04 percent, EGR is 2.21 percent, GED is 2.83 percent, PVR is 3.60 percent, PGR is 0.85 percent, INF is 3.68 percent and the value of standard deviation of REM is 8.28 percent. The value of skewness is -0.14 which describes the series of our sample data of EGR in the selected period is negatively skewed. The values of PED, GED, PVR, PGR, INF and REM are 1.45, 1.07, 1.20, 0.14, 0.66 and 1.21 respectively which shows positive skewness of the data of these variables. As Kurtosis distribution is known as the measure of peakness of any variable, the values of GED and PGR are less than 3 which shows the distribution of these variables is platykurtic. The values of EGR, INF, PED, PVR and REM are greater than 3 which shows the distribution of these variables is leptokurtic. Jarque-Bera is the measure of normal and non-normal variation of the variable. Inside the table of values, PGR indicates normal distribution and the values PED, GED, PVR and REM shows non normal distribution.

**Table 2**  
**Correlation Results**

	<b>EGR</b>	<b>GED</b>	<b>INF</b>	<b>PED</b>	<b>PGR</b>	<b>PVR</b>	<b>REM</b>
<b>EGR</b>	1.00	-0.14	-0.22	-0.09	0.41	-0.26	-0.25
<b>GED</b>		1.00	-0.06	0.97	-0.81	-0.08	0.81
<b>INF</b>			1.00	-0.04	0.06	-0.08	-0.03
<b>PED</b>				1.00	-0.73	-0.15	0.80
<b>PGR</b>					1.00	0.09	-0.78
<b>PVR</b>						1.00	-0.15
<b>REM</b>							1.00

Correlation is a statistical measure of the relationship between variables, with a coefficient ranging from -1 (perfect negative correlation) to 1 (perfect positive correlation). A coefficient of 0 indicates no relationship between the variables. It can examine the nature and intensity of each determinant's relationship to PED in the correlation analyses for Pakistani data provided in Table 2.

PED has a strong negative correlation of -0.73 with PGR, indicating a strong negative relationship between PED and PGR. PED has a moderate negative correlation of -0.15 with



PVR, suggesting a moderate negative relationship between PED and PVR. PED has a strong positive correlation of 0.80 with REM indicating a strong positive relationship between PED and REM. PED has a weak negative correlation of -0.04 with INF, indicating a weak negative relationship between PED and INF. PED has a strong positive correlation of 0.97 with GED, implying a strong positive relationship between PED and GED. In conclusion, PED has no significant correlation with ING and EGR however, strong correlation is found on account of GED and PGR.

**Empirical Analyses**

In empirical analysis, data is often categorized based on risk factors. This involves dividing the data into groups with specific characteristics. In each group, the likelihood of events of interest is estimated and their associations are tested. This study discusses three specific tests: unit root test, cointegration bound test, and autoregressive distributed lag (ARDL) test.

**Unit Root Test**

The unit root test checks for non-stationarity in a time series, indicating a long-term trend instead of a stable mean. Table 3 uses Augmented Dickey-Fuller tests to check for stationarity. Some variables are stationary as-is, while others require first differencing to become stationary.

**Table 3**  
**ADF Test Results**

<b>Variable</b>	<b>Computed t statistic</b>	<b>Prob.</b>	<b>Conclusion</b>
<b>PED</b>	-8.65	0.00	I(1)
<b>EGR</b>	-4.78	0.00	I(0)
<b>GED</b>	-5.81	0.00	I(1)
<b>PVR</b>	-3.06	0.03	I(0)
<b>PGR</b>	-5.51	0.00	I(1)
<b>INF</b>	-3.11	0.03	I(0)
<b>REM</b>	-3.34	0.02	I(0)

**Bounds Test of Cointegration**

The Bounds Test of Cointegration examines long-term relationships between variables. It uses an Error Correction Model (ECM) with both variables and their initial differences. If  $H_0$  of no long-term connection (coefficient on the first difference is zero) is rejected, it signifies cointegration between the variables.

**Table 4**  
**Results of ARDL Bound Test**

<b>Test Statistic</b>	<b>Value</b>	<b>k</b>
<b>F-statistic</b>	3.105312	6
<b>Critical Bound Value at I(1) is 2.93</b>		

The ARDL Bound Test suggests potential long-term connections between Cointeq and the six independent variables based on sample data. We can reject the  $H_0$  of no long-term relationship, as the computed F-statistic of 3.105312 exceeds the I(1) bound of 2.93 at a 5 percent significance level.

**Table 5**  
**Long Run results of Model**  
**Dependent variable (PED)**

<b>Variable</b>	<b>Coefficient</b>	<b>Std. Error</b>	<b>t-statistics</b>	<b>Prob.</b>
<b>EGR</b>	0.80	0.33	2.44	0.02
<b>GED</b>	4.11	1.39	2.96	0.00
<b>PVR</b>	0.19	0.10	1.78	0.09
<b>PGR</b>	-0.91	0.36	-2.54	0.02
<b>INF</b>	0.08	0.06	1.31	0.21
<b>REM</b>	-0.04	0.06	-0.73	0.48
<b>C</b>	-16.29	7.45	-2.19	0.04

In this regression analysis the coefficient of 0.80 suggests that a one-unit increase in EGR corresponds to about a 0.80 unit increase in PED. This relationship is statistically significant at the 0.05 significance level. The coefficient of 4.11 suggests that a one-unit increase in GED leads to 4.11 unit increase in PED. The t-statistic of 2.96 and a p-value of 0.007 indicate that this relationship is statistically significant.

To allocate funds for the education sector is quite crucial for better state of a country. In the world of today, public sector is least sufficient in providing education to the country nationals. Therefore, a meaningful partnership of public and private sector is indeed needed to settle the issue of educational needs of the citizens, as highlighted by Khalid (2016). The results indicate positive association of public and private expenditure in education sector of Pakistan. Earlier, the importance of this sectoral bonding is highlighted by Hassan et al. (2011), Maluleke et al. (2023). The findings thereby support the Maluleke et al. (2023), Awan and Waqas (2019), Irfan (2015), Khalid (2016).

A one-unit increase in PVR is associated with an increase of 0.19 unit in PED. However, the p-value of 0.09 suggests that this relationship is statistically significant at 10 percent.

The coefficient of -0.91 suggests that a one-unit increase in PGR is to decrease PED by 0.91 unit. The t-statistic of -2.54 and a p-value of 0.02 indicate that this relationship is statistically significant. A one-unit increase in INF is associated with an approximate 0.08 unit increase in PED. However, the p-value of 0.21 suggests that this relationship is not statistically significant. The coefficient of -0.04 suggests that a one-unit increase in REM is to cause 0.04-unit decrease in PED. The t-statistic of -0.73 and a p-value of 0.48 indicate that this relationship is not statistically significant. The constant term represents the intercept of the regression equation, which is -16.29. The t-statistic of -2.19 and a p-value of 0.04 indicate that the intercept is statistically significant.

**Table 6**  
**Short Run Analyses**  
**Dependent variable (PED)**

<b>Variable</b>	<b>Coefficient</b>	<b>Std. Error</b>	<b>t-statistics</b>	<b>Prob.</b>
<b>D(EGR)</b>	0.24	0.06	4.11	0.0005
<b>D(EGR(-1))</b>	-0.13	0.08	-1.66	0.11
<b>D(GED)</b>	0.79	0.57	1.37	0.19
<b>D(GED(-1))</b>	-0.99	0.48	-2.06	0.05
<b>D(PVR)</b>	0.07	0.09	0.83	0.42
<b>D(PVR(-1))</b>	-0.09	0.12	-0.76	0.46
<b>D(PVR(-2))</b>	-0.15	0.09	-1.72	0.10
<b>D(PGR)</b>	-0.58	0.29	-1.10	0.06
<b>D(INF)</b>	0.03	0.03	0.92	0.37
<b>D(INF(-1))</b>	-0.06	0.03	-2.02	0.06
<b>D(REM)</b>	0.15	0.06	2.26	0.03

<b>D(REM(-1))</b>	0.17	0.20	0.85	0.40
<b>D(REM(-2))</b>	0.30	0.17	1.81	0.08
<b>CointEq(-1)</b>	-0.64	0.20	-3.14	0.00

The computation of short run results confirm that one-unit increase in the first-differenced GED is associated with a 0.79 unit increase in PED. However, the t-statistic is 1.37, and the p-value is 0.19, indicating that this relationship is not statistically significant. The coefficient for the first-differenced lagged GED is -0.99, implying that a one-unit increase in GED in the previous period is associated with a 0.99 unit decrease in PED. Therefore, government and private sector education expenditure are not complementing each other. The findings on PVR are found insignificant in each category of lags expect that of 2 years. Moreover, PGR and REM are found to significantly influence PED at first difference likewise INF at one year lag. REM is also found to have significant positive impact on PED at 2 years lag. The lagged value of the coefficient of error term of -0.64 with t-statistic of -3.14 and a very low p-value of 0.00 indicates that disequilibrium is adjusted at 64 percent.

The conclusion drawn is that; GED is found to complement PED in long run however in short run both are found substituting one another, respectively.

**Sensitivity Analyses**

Sensitivity analyses assess a reliability of study's findings and robustness. It forecasts outcomes in scenarios where key predictions may not hold, helping to evaluate associated risks and input value dependence. This study examines test reliability for models, including Heteroskedasticity (ARCH), Ramsey RESET and Heteroskedasticity (Breusch-Pagan-Godfrey) tests, along with Histogram-Normality and CUSUM tests.

ARCH models analyze historical volatility and forecast future volatility in time series data for risk estimation in finance, providing a more realistic market volatility model.

**Table 7**  
**Sensitivity Analyses**

<b>Test Type</b>	<b>F-statistic</b>	<b>Conclusion</b>
<b>Heteroskedasticity Test:</b> <b>ARCH</b>	0.09(0.76)	No issue of Heteroskedasticity
<b>Ramsey RESET Test</b>	7.61(0.00)	Correctly specified
<b>Histogram-Normality Test</b>	0.74(0.68)	Residuals are normally distributed

The computed F-statistic is 0.09 while testing  $H_0$  of no conditional heteroscedasticity thus suggesting insufficient evidence to reject the  $H_0$  of no conditional heteroscedasticity. The Ramsey RESET test checks for functional form misspecification in regression models. It tests if the model is correctly specified  $H_0$ , and the alternative hypothesis is that the model omits variables or improperly specifies functional forms. The F-statistic 7.61 suggests that adding the squared term improves the model's explanatory power significantly. This indicates that the model is correctly specified. A normality test is typically conducted to assess if sample data, drawn from a population with a normally distributed distribution, follows a normal pattern. This helps ensure that the survey data is evenly dispersed. The model appears unbiased with close-to-zero mean residuals, showing a slight left skew and leptokurtic behavior (kurtosis > 3). The largest residual is 0.732, the smallest is -0.871, and the low standard deviation (0.314) suggests low variability. The Jarque-Bera test (p-value = 0.68) doesn't reject the possibility of residuals following a normal distribution ( $p > 0.05$ ).

**CUSUM & CUSUM Square Test**

CUSUM is a diagnostic tool used to assess the accuracy of both long-term and short-term parameters. This assessment is made based on the placement of the CUSUM graph

within the 5 percent significance bound. Similarly, The CUSUM Squared test is employed to detect structural breaches, and the null hypothesis suggests that the coefficients remain structurally stable. The CUSUM Squared test did not reveal any signs of structural instability in either model. When the CUSUM squared sequences fall within the critical range, the null hypothesis is rejected, indicating that the coefficients in both models are consistent and stable.

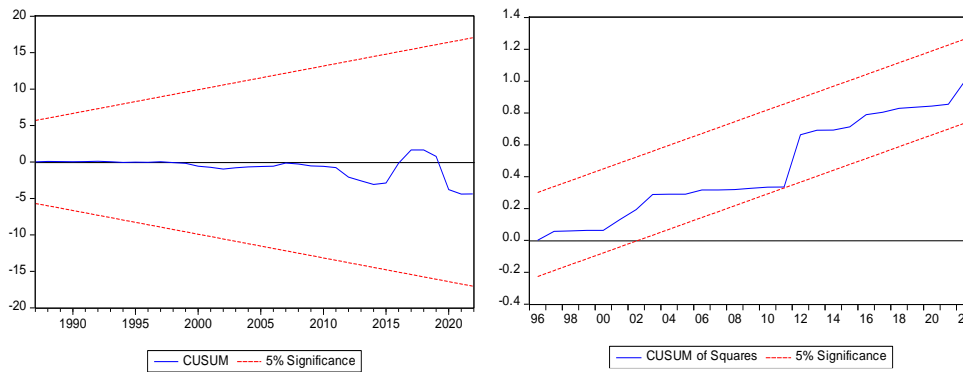


Figure 1 CUSUM & CUSUM Square Test

### Conclusion and Policy Recommendation

This research centered on assessing the sustainability of the PPS with a specific focus on the education sectors in Pakistan. The study analyzed time series data spanning from 1980 to 2022 and applied the ARDL technique for data estimation. In the prescribed model, the ARDL analysis revealed that the long-run coefficient of the PED was statistically affected by GED. The positive sign of coefficient indicated complimentary relationship of public and private investment in education sector of Pakistan. In the short term, there is a statistically significant relationship between PED and GED, however only at lag of one year. The negative short run coefficient indicated substitutability of the two. This suggests that short-term increases in GED are associated with short-term decreases in PED.

In the model, the ARDL long-run analysis revealed a significant relationship between all the chosen variables with the exception of INF and REM.

The findings illustrate the strong synergy between Public-Private partnership of government and private sector on the education side. Thus, there is a need to understand the critical role of PPS in driving economic growth, with a specific focus on Pakistan's educational domain. These results emphasize the significance of PPS in fostering economic development, particularly within Pakistan's education sector. The research strongly advocates to promote allocation of investment capital in education sector of Pakistan altogether by the side of public and private sector.

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